

**CSIE30600/CSIEB0290**  
**Database Systems**

**Lecture 11:**  
**Relational Database**  
**Design II**

## Outline

- Designing a Set of Relations
- Properties of Relational Decompositions
- Algorithms for Relational Database Schema
- Multivalued Dependencies and Fourth Normal Form
- Join Dependencies and Fifth Normal Form
- Inclusion Dependencies
- Other Dependencies and Normal Forms

## Relational Synthesis

- Designing database using **relational synthesis** (Bottom-up Design):
  - Assumes that all possible functional dependencies are known.
  - First constructs a minimal set of FDs
  - Then applies algorithms that construct a target set of 3NF or BCNF relations.
  - Additional criteria may be needed to ensure the *set of relations* in a relational database are satisfactory.

## Design Goals

- **Lossless join property** (a must)
  - Algorithm 15.3 tests for general losslessness.
- **Dependency preservation property**
  - Algorithm 15.5 decomposes a relation into BCNF components by sacrificing the dependency preservation.
- **Additional normal forms**
  - 4NF (based on multi-valued dependencies)
  - 5NF (based on join dependencies)

## Relational Decompositions (1)

- **Universal Relation Schema:**
  - A relation schema  $R = \{A_1, A_2, \dots, A_n\}$  that includes **all** the attributes of the database.
- **Universal relation assumption:**
  - Every attribute name is unique.

## Relational Decompositions (2)

- **Relational Decomposition:**
  - The process of decomposing the universal relation schema  $R$  into a set of relation schemas  $D = \{R_1, R_2, \dots, R_m\}$  that will become the relational database schema by using the functional dependencies.
- **Attribute preservation condition:**
  - Each attribute in  $R$  will appear in **at least one** relation schema  $R_i$  in the decomposition so that no attributes are “lost”.

## Relational Decompositions (3)

- Another goal of decomposition is to have each individual relation  $R_i$  in the decomposition  $D$  be in **BCNF** or **3NF**.
- Additional properties of decomposition are needed to prevent from generating spurious tuples

## Dependency Preservation (1)

- **Definition:** Given a set of dependencies  $F$  on  $R$ , the **projection** of  $F$  on  $R_i$ , denoted by  $\pi_{R_i}(F)$  where  $R_i$  is a subset of  $R$ , is the set of dependencies  $X \rightarrow Y$  in  $F^+$  such that the **attributes** in  $X \cup Y$  are **all** contained in  $R_i$ .
- Hence, the projection of  $F$  on each relation schema  $R_i$  is the set of functional dependencies in  $F^+$  (the closure of  $F$ ) such that **all** their **left- and right-hand-side attributes** are in  $R_i$ .

## Dependency Preservation (2)

- Dependency Preservation Property:
  - A decomposition  $D = \{R_1, R_2, \dots, R_m\}$  of  $R$  is **dependency-preserving** with respect to  $F$  if the **union** of the **projections** of  $F$  on each  $R_i$  in  $D$  is equivalent to  $F$ ; that is
 
$$((\pi_{R_1}(F)) \cup \dots \cup (\pi_{R_m}(F)))^+ = F^+$$
- Claim 1:
  - It is **always** possible to find a **dependency-preserving decomposition**  $D$  with respect to  $F$  such that each relation  $R_i$  in  $D$  is in **3NF**.

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## Testing for Dependency Preservation

- To check if a dependency  $\alpha \rightarrow \beta$  is preserved in a decomposition of  $R$  into  $R_1, R_2, \dots, R_m$  we apply the following test (with attribute closure done with respect to  $F$ )
  - $result = \alpha$ 
    - while** (changes to  $result$ ) do
      - for each**  $R_i$  in the decomposition
 
$$t = (result \cap R_i)^+ \cap R_i$$

$$result = result \cup t$$
  - If  $result$  contains all attributes in  $\beta$ , then the functional dependency  $\alpha \rightarrow \beta$  is preserved.

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## Testing for Dependency Preservation

- We apply the test on all dependencies in  $F$  to check if a decomposition is dependency preserving
- This procedure takes polynomial time, instead of the exponential time required to compute  $F^+$  and  $(F_1 \cup F_2 \cup \dots \cup F_n)^+$

## Example

- $R = (A, B, C)$   
 $F = \{A \rightarrow B, B \rightarrow C\}$   
Key =  $\{A\}$
- $R$  is not in BCNF
- Decomposition  $R_1 = (A, B), R_2 = (B, C)$ 
  - $R_1$  and  $R_2$  in BCNF
  - Dependency preserving
  - Lossless-join decomposition (next slide)

## Lossless (Non-additive) Join

- **Definition:** Lossless join property: a decomposition  $D = \{R_1, R_2, \dots, R_m\}$  of  $R$  has the **lossless (nonadditive) join property** with respect to the set of dependencies  $F$  on  $R$  if, for every relation state  $r$  of  $R$  that satisfies  $F$ , the following holds, where  $*$  is the **natural join** of all the relations in  $D$ :

$$* (\pi_{R_1}(r), \dots, \pi_{R_m}(r)) = r$$

- **Note:** The word loss in lossless refers to **loss of information**, not to loss of tuples. In fact, for “loss of information” a better term is “**addition of spurious information**”

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## Testing Lossless Join (1)

- **Algorithm 15.3: Testing for Lossless Join Property**
  - **Input:** A universal relation  $R$ , a decomposition  $D = \{R_1, R_2, \dots, R_m\}$  of  $R$ , and a set  $F$  of functional dependencies.
  - 1. Create an initial **matrix  $S$**  with **one row  $i$**  for each relation  $R_i$  in  $D$ , and **one column  $j$**  for each **attribute  $A_j$**  in  $R$ .
  - 2. Set  **$S(i, j) := b_{ij}$**  for all matrix entries. (\* each  $b_{ij}$  is a distinct symbol associated with indices  $(i, j)$  \*).
  - 3. For each row  $i$  representing relation schema  $R_i$ 
    - {for each column  $j$  representing attribute  $A_j$
    - {if (relation  $R_i$  includes attribute  $A_j$ ) then
    - set  **$S(i, j) := a_j$** ; }; (\* each  $a_j$  is a distinct symbol associated with index  $(j)$  \*)

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## Testing Lossless Join (2)

4. Repeat the following loop until no changes to  $S$ 
  - {for each functional dependency  $X \rightarrow Y$  in  $F$
  - {for all rows in  $S$  which have the same symbols in the columns corresponding to attributes in  $X$ 
    - { make the symbols in each column that correspond to an attribute in  $Y$  be the same in all these rows as follows:
      - If any of the rows has an "a" symbol for the column, set the other rows to that same "a" symbol in the column.
      - If no "a" symbol exists for the attribute in any of the rows, choose one of the "b" symbols that appear in one of the rows for the attribute and set the other rows to that same "b" symbol in the column ;
5. If a row is made up entirely of "a" symbols, then the decomposition has the lossless join property; otherwise it does not.

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## Testing Lossless Join Example (1)

Lossless (nonadditive) join test for  $n$ -ary decompositions.

(a) Case 1: Decomposition of EMP\_PROJ into EMP\_PROJ1 and EMP\_LOCS fails test.

(b) A decomposition of EMP\_PROJ that has the lossless join property.

- (a)  $R = \{Ssn, Ename, Pnumber, Pname, Plocation, Hours\}$   $D = \{R_1, R_2\}$   
 $R_1 = EMP\_LOCS = \{Ename, Plocation\}$   
 $R_2 = EMP\_PROJ1 = \{Ssn, Pnumber, Hours, Pname, Plocation\}$

$F = \{Ssn \twoheadrightarrow Ename; Pnumber \twoheadrightarrow \{Pname, Plocation\}; \{Ssn, Pnumber\} \twoheadrightarrow Hours\}$

	Ssn	Ename	Pnumber	Pname	Plocation	Hours
$R_1$	$b_{11}$	$a_2$	$b_{13}$	$b_{14}$	$a_5$	$b_{16}$
$R_2$	$a_1$	$b_{22}$	$a_3$	$a_4$	$a_5$	$a_6$

(No changes to matrix after applying functional dependencies)

- (b)
- | EMP          |
|--------------|
| Ssn    Ename |
- | PROJECT                       |
|-------------------------------|
| Pnumber    Pname    Plocation |
- | WORKS_ON                |
|-------------------------|
| Ssn    Pnumber    Hours |

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## Testing Lossless Join Example (2)

Lossless (nonadditive) join test for n-ary decompositions.

(c) Case 2: Decomposition of EMP\_PROJ into EMP, PROJECT, and WORKS\_ON satisfies test.

(c)  $R = \{\text{Ssn, Ename, Pnumber, Pname, Plocation, Hours}\}$   $D = \{R_1, R_2, R_3\}$   
 $R_1 = \text{EMP} = \{\text{Ssn, Ename}\}$   
 $R_2 = \text{PROJ} = \{\text{Pnumber, Pname, Plocation}\}$   
 $R_3 = \text{WORKS\_ON} = \{\text{Ssn, Pnumber, Hours}\}$

$F = \{\text{Ssn} \twoheadrightarrow \text{Ename}; \text{Pnumber} \twoheadrightarrow \{\text{Pname, Plocation}\}; \{\text{Ssn, Pnumber}\} \twoheadrightarrow \text{Hours}\}$

	Ssn	Ename	Pnumber	Pname	Plocation	Hours
$R_1$	$a_1$	$a_2$	$b_{13}$	$b_{14}$	$b_{15}$	$b_{16}$
$R_2$	$b_{21}$	$b_{22}$	$a_3$	$a_4$	$a_5$	$b_{26}$
$R_3$	$a_1$	$b_{32}$	$a_3$	$b_{34}$	$b_{35}$	$a_6$

(Original matrix S at start of algorithm)

	Ssn	Ename	Pnumber	Pname	Plocation	Hours
$R_1$	$a_1$	$a_2$	$b_{13}$	$b_{14}$	$b_{15}$	$b_{16}$
$R_2$	$b_{21}$	$b_{22}$	$a_3$	$a_4$	$a_5$	$b_{26}$
$R_3$	$a_1$	$b_{32} a_2$	$a_3$	$b_{34} a_4$	$b_{35} a_5$	$a_6$

(Matrix S after applying the first two functional dependencies; last row is all "a" symbols so we stop)

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## Testing Lossless Join on Binary D

- **Testing Binary Decompositions for Lossless Join Property**
  - **Binary Decomposition:** Decomposition of a relation R into two relations.
  - **PROPERTY LJ<sub>1</sub> (lossless join test for binary decompositions):** A decomposition  $D = \{R_1, R_2\}$  of R has the lossless join property with respect to a set of functional dependencies F on R *if and only if* either
    - The FD  $((R_1 \cap R_2) \rightarrow (R_1 - R_2))$  is in  $F^+$ , or
    - The FD  $((R_1 \cap R_2) \rightarrow (R_2 - R_1))$  is in  $F^+$ .

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## Example

- $R = (A, B, C)$   $F = \{A \rightarrow B, B \rightarrow C\}$ 
  - Can be decomposed in two different ways
- $R_1 = (A, B), R_2 = (B, C)$ 
  - Lossless-join decomposition:  
 $R_1 \cap R_2 = \{B\}$  and  $B \rightarrow C$  ( $R_2 - R_1$ )
  - Dependency preserving
- $R_1 = (A, B), R_2 = (A, C)$ 
  - Lossless-join decomposition:  
 $R_1 \cap R_2 = \{A\}$  and  $A \rightarrow B$  ( $R_1 - R_2$ )
  - Not dependency preserving  
(cannot check  $B \rightarrow C$  w/o computing  $R_1 \bowtie R_2$ )

## Successive Lossless Join

- **Successive Lossless Join Decomposition:**
  - **Claim 2 (Preservation of non-additivity in successive decompositions):**
    - If a decomposition  $D = \{R_1, R_2, \dots, R_m\}$  of  $R$  has the lossless (non-additive) join property with respect to a set of functional dependencies  $F$  on  $R$ ,
    - and if a decomposition  $D_1 = \{Q_1, Q_2, \dots, Q_k\}$  of  $R_1$  has the lossless (non-additive) join property with respect to the projection of  $F$  on  $R_1$ ,
      - then the decomposition  $D_2 = \{R_1, R_2, \dots, R_{i-1}, Q_1, Q_2, \dots, Q_k, R_{i+1}, \dots, R_m\}$  of  $R$  has the non-additive join property with respect to  $F$ .

## Algorithms for RDB Design - Finding Minimal Cover

- Algorithm 15.2: Find a minimal cover  $F$  for a set of FDs  $E$
- 1.  $F := E$
- 2. Replace each FD  $X \rightarrow \{A_1, A_2, \dots, A_n\}$  in  $F$  by  $X \rightarrow A_1, X \rightarrow A_2, \dots, X \rightarrow A_n$  (\* convert  $F$  so that all RHS has only one attribute \*)
- 3. For each  $X \rightarrow A$  in  $F$  (\* remove extraneous attributes in LHS \*)

For each attribute  $B \in X$

if  $\{F - \{X \rightarrow A\}\} \cup \{(X - \{B\}) \rightarrow A\} \equiv F$

replace  $X \rightarrow A$  with  $(X - \{B\}) \rightarrow A$  in  $F$

1. For each  $X \rightarrow A$  in  $F$  (\* remove extraneous FD \*)
  - if  $F - \{X \rightarrow A\}$  is equivalent to  $F$
  - remove  $X \rightarrow A$  from  $F$

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## Computing a Minimal Cover

- $R = (A, B, C)$   
 $F = \{A \rightarrow BC, B \rightarrow C, A \rightarrow B, AB \rightarrow C\}$
- Replace  $A \rightarrow BC$  by  $A \rightarrow B$  and  $A \rightarrow C$ 
  - Set is now  $\{A \rightarrow B, A \rightarrow C, B \rightarrow C, AB \rightarrow C\}$
- $A$  is extraneous in  $AB \rightarrow C$ 
  - Set is now  $\{A \rightarrow B, A \rightarrow C, B \rightarrow C\}$
- $A \rightarrow C$  is extraneous since it can be inferred from  $A \rightarrow B$  and  $B \rightarrow C$
- The minimal cover is:  $\{A \rightarrow B, B \rightarrow C\}$

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## Algorithms for RDB Design - Key Determination

- **Algorithm 15.2(a) Finding a Key K for R Given a set F of Functional Dependencies**

- **Input: A universal relation R and a set of FDs F**

1. Set  $K := R$ ;
2. For each attribute A in K {
  - Compute  $(K - A)^+$  with respect to F;
  - If  $(K - A)^+$  contains **all attributes** in R,
  - then set  $K := K - \{A\}$ ;

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## Relational Synthesis into 3NF

- **Algorithm 15.4: Relational Synthesis into 3NF with Dependency Preservation and Lossless (Non-Additive) Join Property**

- **Input: A universal relation R and a set of FDs F**

1. Find a minimal cover G for F (Algorithm 15.2)
  2. For each LHS X of a FD in G, create a schema in D with attributes  $\{X \cup \{A_1\} \cup \{A_2\} \dots \cup \{A_k\}\}$ , where  $X \rightarrow A_1, X \rightarrow A_2, \dots, X \rightarrow A_k$  are the only dependencies in G with X as LHS (X is the key of this relation).
  3. If none of the relation schemas in D contains a key of R, then create one more relation schema in D that contains attributes that form a key of R. (Use Algorithm 15.2(a) to find the key of R)
  4. Eliminate redundant relations (subsumed by others)
- **Claim 3: Every relation schema created by Algorithm 15.4 is in 3NF.**

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## Relational Decomposition into BCNF

- **Algorithm 15.5:** Relational Decomposition into **BCNF** with **Lossless (non-additive) join** property
  - **Input:** A universal relation  $R$  and a set of FDs  $F$
- 1. Set  $D := \{R\}$ ;
- 2. While (there is a schema  $Q$  in  $D$  that is not in BCNF) do {
  - choose a schema  $Q$  in  $D$  that is **not in BCNF**;
  - find a **FD  $X \rightarrow Y$**  in  $Q$  that **violates BCNF**;
  - replace  $Q$**  in  $D$  by two schemas  **$(Q - Y)$**  and  **$(X \cup Y)$** ;
- };

*Assumption: No null values are allowed for the join attributes.*

## Example of BCNF Decomposition

- $R = (A, B, C)$   
 $F = \{A \rightarrow B, B \rightarrow C\}$   
 Key =  $\{A\}$
- $R$  is not in BCNF ( $B \rightarrow C$  but  $B$  is not a superkey)
- Decomposition
  - $R_1 = (B, C)$
  - $R_2 = (A, B)$



## Example of BCNF Decomposition

- Original relation  $R$  and FD  $F$ 

$$R = (\text{branch\_name}, \text{branch\_city}, \text{assets},$$

$$\text{customer\_name}, \text{loan\_number}, \text{amount})$$

$$F = \{\text{branch\_name} \rightarrow \text{assets branch\_city}$$

$$\text{loan\_number} \rightarrow \text{amount branch\_name}\}$$

$$\text{Key} = \{\text{loan\_number}, \text{customer\_name}\}$$
- Decomposition
  - $R_1 = (\text{branch\_name}, \text{branch\_city}, \text{assets})$
  - $R_2 = (\text{branch\_name}, \text{customer\_name}, \text{loan\_number}, \text{amount})$
  - $R_3 = (\text{branch\_name}, \text{loan\_number}, \text{amount})$
  - $R_4 = (\text{customer\_name}, \text{loan\_number})$
- Final decomposition:  $R_1, R_3, R_4$

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## BCNF and Dependency Preservation

- It is **not always possible** to get a BCNF decomposition that is **dependency preserving**
- $R = (J, K, L)$ 

$$F = \{JK \rightarrow L, L \rightarrow K\}$$

Two candidate keys =  $JK$  and  $JL$
- $R$  is not in BCNF
- Any decomposition of  $R$  will fail to preserve
 
$$JK \rightarrow L$$

This implies that testing for  $JK \rightarrow L$  requires a join

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## Example

- Relation schema:  
 $cust\_banker\_branch = (customer\_id, employee\_id, branch\_name, type)$
- The functional dependencies for this relation schema are:  
 $customer\_id, employee\_id \rightarrow branch\_name, type$   
 $employee\_id \rightarrow branch\_name$
- The **for** loop generates:  
 $(customer\_id, employee\_id, branch\_name, type)$   
 It then generates  
 $(employee\_id, branch\_name)$   
 but does not include it in the decomposition because it is a subset of the first schema.

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## Comparison of BCNF and 3NF

- It is **always** possible to **decompose** a relation into a set of relations that are in **3NF** st:
  - the decomposition is **lossless**
  - the **dependencies** are **preserved**
- It is **always** possible to decompose a relation into a set of relations that are in **BCNF** st:
  - the decomposition is **lossless**
  - it **may not** be possible to preserve dependencies.

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## Discussion of Normalization Algorithms

- Problems:
  - The database designer must first specify *all* the relevant functional dependencies among the database attributes.
  - These algorithms are *not deterministic* in general.
  - It is *not always possible* to find a decomposition into relation schemas that *preserves dependencies* and allows each relation schema in the decomposition to be *in BCNF* (instead of 3NF as in Algorithm 15.4).

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## Summary of Algorithms

**Table 15.1** Summary of the Algorithms Discussed in This Chapter

Algorithm	Input	Output	Properties/Purpose	Remarks
15.1	An attribute or a set of attributes $X$ , and a set of FDs $F$	A set of attributes in the closure of $X$ with respect to $F$	Determine all the attributes that can be functionally determined from $X$	The closure of a key is the entire relation
15.2	A set of functional dependencies $F$	The minimal cover of functional dependencies	To determine the minimal cover of a set of dependencies $F$	Multiple minimal covers may exist—depends on the order of selecting functional dependencies
15.2a	Relation schema $R$ with a set of functional dependencies $F$	Key $K$ of $R$	To find a key $K$ (that is a subset of $R$ )	The entire relation $R$ is always a default superkey
15.3	A decomposition $D$ of $R$ and a set $F$ of functional dependencies	Boolean result: yes or no for nonadditive join property	Testing for nonadditive join decomposition	See a simpler test NJB in Section 14.5 for binary decompositions
15.4	A relation $R$ and a set of functional dependencies $F$	A set of relations in 3NF	Nonadditive join and dependency-preserving decomposition	May not achieve BCNF, but achieves <i>all</i> desirable properties and 3NF
15.5	A relation $R$ and a set of functional dependencies $F$	A set of relations in BCNF	Nonadditive join decomposition	No guarantee of dependency preservation
15.6	A relation $R$ and a set of functional and multivalued dependencies	A set of relations in 4NF	Nonadditive join decomposition	No guarantee of dependency preservation

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## Design Goals

- **Goals** for a relational database design is:
  - **BCNF**.
  - Lossless join.
  - Dependency preservation.
- If we cannot achieve all, we accept one of
  - Lack of dependency preservation
  - Redundancy due to use of **3NF**
- Interestingly, SQL does not provide a direct way of specifying functional dependencies other than superkeys. (Can specify FDs using assertions, but they are expensive to test)
- Even if we had a dependency preserving decomposition, using SQL we would not be able to efficiently test a functional dependency whose left hand side is not a key.

## Multivalued Dependencies (MVDs)

- Let  $R$  be a relation schema and let  $\alpha \subseteq R$  and  $\beta \subseteq R$ .  
The *multivalued dependency*

$$\alpha \twoheadrightarrow \beta$$

holds on  $R$  if in any legal relation  $r(R)$ , for all pairs for tuples  $t_1$  and  $t_2$  in  $r$  such that  $t_1[\alpha] = t_2[\alpha]$ , there exist tuples  $t_3$  and  $t_4$  in  $r$  s.t.:

$$\begin{aligned} t_1[\alpha] &= t_2[\alpha] = t_3[\alpha] = t_4[\alpha] \\ t_3[\beta] &= t_1[\beta] & t_3[R-\beta] &= t_2[R-\beta] \\ t_4[\beta] &= t_2[\beta] & t_4[R-\beta] &= t_1[R-\beta] \end{aligned}$$

## MVD (Cont.)

- Tabular representation of  $\alpha \twoheadrightarrow \beta$

	$\alpha$	$\beta$	$R - \alpha - \beta$
$t_1$	$a_1 \dots a_i$	$a_{i+1} \dots a_j$	$a_{j+1} \dots a_n$
$t_2$	$a_1 \dots a_i$	$b_{i+1} \dots b_j$	$b_{j+1} \dots b_n$
$t_3$	$a_1 \dots a_i$	$a_{i+1} \dots a_j$	$b_{j+1} \dots b_n$
$t_4$	$a_1 \dots a_i$	$b_{i+1} \dots b_j$	$a_{j+1} \dots a_n$

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## Another View of MVD

- Let  $R$  be a relation schema with a set of attributes that are partitioned into 3 nonempty subsets.

$$Y, Z, W$$

- We say that  $Y \twoheadrightarrow Z$  ( $Y$  **multidetermines**  $Z$ ) if and only if for all possible relations  $r (R)$

$$\langle y_1, z_1, w_1 \rangle \in r \text{ and } \langle y_1, z_2, w_2 \rangle \in r$$

then

$$\langle y_1, z_1, w_2 \rangle \in r \text{ and } \langle y_1, z_2, w_1 \rangle \in r$$

- Note that since the behavior of  $Z$  and  $W$  are identical it follows that  $Y \twoheadrightarrow Z$  if  $Y \twoheadrightarrow W$

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## Example

- In our (course, teacher, book) example:  
 $course \twoheadrightarrow teacher$       $course \twoheadrightarrow book$
- The above formal definition is supposed to formalize the notion that given a particular value of  $Y$  (*course*) it has associated with it a set of values of  $Z$  (*teacher*) and a set of values of  $W$  (*book*), and these **two sets** are in some sense **independent** of each other. (next slide)
- Note: If  $Y \rightarrow Z$  then  $Y \twoheadrightarrow Z$ 
  - Indeed we have (in above notation)  $Z_1 = Z_2$   
The claim follows.

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## Example of MVD

	<i>course</i>	<i>teacher</i>	<i>book</i>
t3	database	Avi	DB Concepts
t1	database	Avi	Ullman
t2	database	Hank	DB Concepts
t4	database	Hank	Ullman
	database	Sudarshan	DB Concepts
	database	Sudarshan	Ullman
	operating system	Avi	OS Concepts
	operating system	Avi	Stallings
	operating system	Pete	OS Concepts
	operating system	Pete	Stallings

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## Use of Multivalued Dependencies

- We use MVDs in two ways:
  1. To test relations to determine whether they are legal under a given set of functional and multivalued dependencies
  2. To specify **constraints** on the set of legal relations. We shall thus concern ourselves *only* with relations that satisfy a given set of functional and multivalued dependencies.
- If a relation  $r$  fails to satisfy a given multivalued dependency, we can construct a relations  $r'$  that does satisfy the multivalued dependency by adding tuples to  $r$ .

## Theory of MVDs

- From the definition of multivalued dependency, we can derive the following rule:
  - If  $\alpha \rightarrow \beta$ , then  $\alpha \twoheadrightarrow \beta$

That is, every functional dependency is also a multivalued dependency
- The **closure**  $D^+$  of  $D$  is the set of all functional and multivalued dependencies logically implied by  $D$ .
  - We can compute  $D^+$  from  $D$ , using the formal definitions of functional dependencies and multivalued dependencies.
  - We can manage with such reasoning for very simple MVDs, which seem to be most common in practice
  - For complex dependencies, it is better to reason about sets of dependencies using a system of **inference rules**.

## Inference Rules for FD's and MVD's

- The following set of rules is **sound** and **complete**.

- For FD's:

IR<sub>1</sub> (**Reflexive rule**):

$$\{X \supseteq Y\} \models X \rightarrow Y$$

IR<sub>2</sub> (**Augmentation rule**):

$$\{X \rightarrow Y\} \models XZ \rightarrow YZ$$

IR<sub>3</sub> (**Transitive rule**):

$$\{X \rightarrow Y, Y \rightarrow Z\} \models X \rightarrow Z$$

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## Inference Rules for MVD's

IR<sub>4</sub> (**Complementation rule**):

$$\{X \twoheadrightarrow Y\} \models \{X \twoheadrightarrow (R - (X \cup Y))\}$$

IR<sub>5</sub> (**Multivalued augmentation rule**):

$$\text{if } X \twoheadrightarrow Y \text{ and } W \supseteq Z \text{ then } WX \twoheadrightarrow YZ$$

IR<sub>6</sub> (**Multivalued transitive rule**):

$$\{X \twoheadrightarrow Y, Y \twoheadrightarrow Z\} \models X \twoheadrightarrow (Z - Y)$$

IR<sub>7</sub> (**Replication rule**):

$$\{X \rightarrow Y\} \models X \twoheadrightarrow Y$$

IR<sub>8</sub> (**Coalescence rule**):

$$\text{if } X \twoheadrightarrow Y \text{ and } \exists W \text{ such that } W \cap Y = \emptyset \text{ and } W \twoheadrightarrow Z \text{ and } Y \supseteq Z, \text{ then } X \rightarrow Z$$

- Note that an FD is a special case of MVD.

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## Fourth Normal Form (4NF)

- A relation schema  $R$  is in **4NF** with respect to a set  $D$  of **functional** and **multivalued dependencies** if for all multivalued dependencies in  $D^+$  of the form  $\alpha \twoheadrightarrow \beta$ , where  $\alpha \subseteq R$  and  $\beta \subseteq R$ , at least one of the following hold:
  - $\alpha \twoheadrightarrow \beta$  is trivial (i.e.,  $\beta \subseteq \alpha$  or  $\alpha \cup \beta = R$ )
  - $\alpha$  is a superkey for schema  $R$
- If a relation is in 4NF it is in BCNF  
(Proof: Exercise)

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## Example of 4NF

- (a) The EMP relation with two MVDs:  $ENAME \twoheadrightarrow PNAME$  and  $ENAME \twoheadrightarrow DNAME$ .
- (b) Decomposing the EMP relation into two 4NF relations EMP\_PROJECTS and EMP\_DEPENDENTS.

(a) EMP

ENAME	PNAME	DNAME
Smith	X	John
Smith	Y	Anna
Smith	X	Anna
Smith	Y	John
Brown	W	Jim
Brown	X	Jim
Brown	Y	Jim
Brown	Z	Jim
Brown	W	Joan
Brown	X	Joan
Brown	Y	Joan
Brown	Z	Joan
Brown	W	Bob
Brown	X	Bob
Brown	Y	Bob
Brown	Z	Bob

(b) EMP\_PROJECTS

ENAME	PNAME
Smith	X
Smith	Y
Brown	W
Brown	X
Brown	Y
Brown	Z

EMP\_DEPENDENTS

ENAME	DNAME
Smith	Anna
Smith	John
Brown	Jim
Brown	Joan
Brown	Bob

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## Restriction of Multivalued Dependencies

- The restriction of  $D$  to  $R_i$  is the set  $D_i$  consisting of
  - All functional dependencies in  $D^+$  that include only attributes of  $R_i$
  - All multivalued dependencies of the form
 
$$\alpha \twoheadrightarrow (\beta \cap R_i)$$
 where  $\alpha \subseteq R_i$  and  $\alpha \twoheadrightarrow \beta$  is in  $D^+$

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## 4NF Decomposition Algorithm

```

result := {R};
done := false;
compute D+;
Let Di denote the restriction of D+ to Ri
while (not done)
  if (there is a schema Ri in result that is not in 4NF) then
    begin
      let  $\alpha \twoheadrightarrow \beta$  be a nontrivial MVD that holds on Ri s.t.
         $\alpha \rightarrow R_i$  is not in Di, and  $\alpha \cap \beta = \phi$ ;
      result := (result - Ri)  $\cup$  (Ri -  $\beta$ )  $\cup$  ( $\alpha$ ,  $\beta$ );
    end
  else done := true;

```

(Note: each  $R_i$  is in 4NF, and decomposition is lossless-join)

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## Lossless (Non-additive) Join Decomposition into 4NF Relations

- The relation schemas  $R_1$  and  $R_2$  form a lossless (non-additive) join decomposition of  $R$  with respect to a set  $F$  of functional *and* multivalued dependencies if and only if
  - $(R_1 \cap R_2) \twoheadrightarrow (R_1 - R_2)$
- or by symmetry, if and only if
  - $(R_1 \cap R_2) \twoheadrightarrow (R_2 - R_1)$
- **Proof:** Exercise.

## Decomposition into 4NF relations with non-additive join property

- **Algorithm 15.7: Input:** A universal relation  $R$  and a set of functional and multivalued dependencies  $F$ .
1. Set  $D := \{ R \}$ ;
  2. While there is a relation schema  $Q$  in  $D$  that is not in 4NF do {
    - choose a relation schema  $Q$  in  $D$  that is not in 4NF;
    - find a nontrivial MVD  $X \twoheadrightarrow Y$  in  $Q$  that violates 4NF;
    - replace  $Q$  in  $D$  by two relation schemas  $(Q - Y)$  and  $(X \cup Y)$ ;
- };

## Example

- $R = (A, B, C, G, H, I)$   
 $F = \{ A \twoheadrightarrow B, B \twoheadrightarrow HI, CG \twoheadrightarrow H \}$
- $R$  is not in 4NF since  $A \twoheadrightarrow B$  and  $A$  is not a superkey for  $R$
- Decomposition
  - a)  $R_1 = (A, B)$  ( $R_1$  is in 4NF)
  - b)  $R_2 = (A, C, G, H, I)$  ( $R_2$  is not in 4NF)
  - c)  $R_3 = (C, G, H)$  ( $R_3$  is in 4NF)
  - d)  $R_4 = (A, C, G, I)$  ( $R_4$  is not in 4NF)
- Since  $A \twoheadrightarrow B$  and  $B \twoheadrightarrow HI$ ,  $A \twoheadrightarrow HI$ ,  $A \twoheadrightarrow I$ 
  - e)  $R_5 = (A, I)$  ( $R_5$  is in 4NF)
  - f)  $R_6 = (A, C, G)$  ( $R_6$  is in 4NF)

## Another Example

- $R = (\text{course}, \text{teacher}, \text{book})$   
 $\text{course} \twoheadrightarrow \text{teacher}$        $\text{course} \twoheadrightarrow \text{book}$   
 $R$  is not in 4NF
- $R$  can be decomposed into
  - $R_1 = (\text{course}, \text{teacher})$
  - $R_2 = (\text{course}, \text{book})$
 Both  $R_1$  and  $R_2$  are now in 4NF.

## Join Dependencies and Fifth Normal Form (1)

- A **join dependency (JD)**, denoted by  $JD(R_1, R_2, \dots, R_n)$ , specified on relation schema  $R$ , specifies a constraint on the states  $r$  of  $R$ .
  - The constraint states that every legal state  $r$  of  $R$  should have a non-additive join decomposition into  $R_1, R_2, \dots, R_n$ ; that is, for every such  $r$  we have

$$* (\pi_{R_1}(r), \pi_{R_2}(r), \dots, \pi_{R_n}(r)) = r$$

**Note:** an MVD is a special case of a JD where  $n = 2$ .

- A join dependency  $JD(R_1, R_2, \dots, R_n)$ , specified on relation schema  $R$ , is a **trivial JD** if one of the relation schemas  $R_i$  in  $JD(R_1, R_2, \dots, R_n)$  is equal to  $R$ .

## Join Dependencies

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## JD and Fifth Normal Form

- A relation schema  $R$  is in **fifth normal form (5NF)** (or **Project-Join Normal Form (PJNF)**) with respect to a set  $F$  of functional, multivalued, and join dependencies if,
  - for every nontrivial join dependency  $JD(R_1, R_2, \dots, R_n)$  in  $F^+$  (that is, implied by  $F$ ), every  $R_i$  is a superkey of  $R$ .

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## Example of 5NF Normalization

- (c) The relation SUPPLY with no MVDs is in 4NF but not in 5NF if it has the  $JD(R_1, R_2, R_3)$ .  
 (d) Decomposing the relation SUPPLY into the 5NF relations  $R_1, R_2, R_3$ .

(c) SUPPLY

Sname	Part_name	Proj_name
Smith	Bolt	ProjX
Smith	Nut	ProjY
Adamsky	Bolt	ProjY
Walton	Nut	ProjZ
Adamsky	Nail	ProjX
Adamsky	Bolt	ProjX
Smith	Bolt	ProjY

(d)  $R_1$ 

Sname	Part_name
Smith	Bolt
Smith	Nut
Adamsky	Bolt
Walton	Nut
Adamsky	Nail

 $R_2$ 

Sname	Proj_name
Smith	ProjX
Smith	ProjY
Adamsky	ProjY
Walton	ProjZ
Adamsky	ProjX

 $R_3$ 

Part_name	Proj_name
Bolt	ProjX
Nut	ProjY
Bolt	ProjY
Nut	ProjZ
Nail	ProjX

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## Inclusion Dependencies (1)

- An **inclusion dependency**  $R.X < S.Y$  between two sets of attributes ( $X$  of relation schema  $R$ , and  $Y$  of relation schema  $S$ ) specifies the constraint that, at any specific time when  $r$  is a relation state of  $R$  and  $s$  a relation state of  $S$ , we must have

$$\pi_X(r(R)) \supseteq \pi_Y(s(S))$$

- **Note:**
  - The  $\supseteq$  (subset) relationship does not necessarily have to be a proper subset.
  - The sets of attributes on which the inclusion dependency is specified— $X$  of  $R$  and  $Y$  of  $S$ —must have the same number of attributes.
  - In addition, the domains for each pair of corresponding attributes should be compatible.

## Inclusion Dependencies (2)

- **Objective of Inclusion Dependencies:**
  - To formalize two types of **interrelational constraints** which cannot be expressed using F.D.s or MVDs:
    - **Referential integrity** constraints
    - **Class/subclass** relationships
- **Inclusion dependency inference rules**
  - IDIR<sub>1</sub> (reflexivity):  $R.X < R.X$ .
  - IDIR<sub>2</sub> (attribute correspondence): If  $R.X < S.Y$ 
    - where  $X = \{A_1, A_2, \dots, A_n\}$  and  $Y = \{B_1, B_2, \dots, B_n\}$  and  $A_i$  Corresponds-to  $B_i$ , then  $R.A_i < S.B_i$  for  $1 \leq i \leq n$ .
  - IDIR<sub>3</sub> (transitivity): If  $R.X < S.Y$  and  $S.Y < T.Z$ , then  $R.X < T.Z$ .



## Domain-Key Normal Form(DKNF)

- **Definition:**

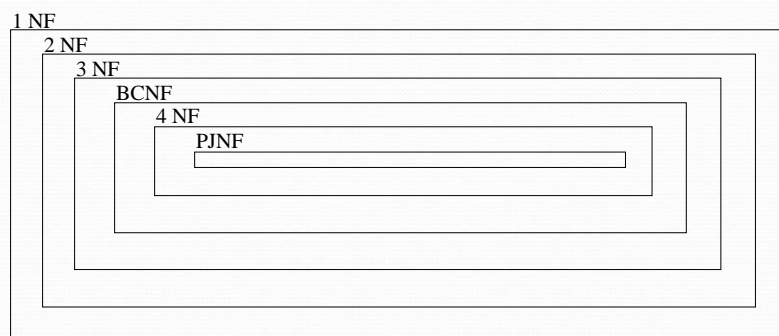
- A relation schema is said to be in **DKNF** if all constraints and dependencies that should hold on the valid relation states can be enforced simply by enforcing the domain constraints and key constraints on the relation.
- The idea is to specify (theoretically, at least) the “**ultimate normal form**” that takes into account all possible types of dependencies and constraints. .
- For a relation in DKNF, it becomes very straightforward to enforce all database constraints by simply checking that each attribute value in a tuple is of the appropriate domain and that every key constraint is enforced.
- The practical utility of DKNF is limited

## Note on Higher Normal Forms

- 5NF and DKNF are rarely used
- Problem with these generalized constraints: are hard to reason with, and no set of sound and complete set of inference rules exists.

## Levels of Normalization

- The relationship between various normal forms:



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## Overall Database Design Process

- We have assumed schema  $R$  is given
  - $R$  could have been generated when converting ER-diagram to a set of tables.
  - $R$  could have been a single relation containing *all* attributes that are of interest (called **universal relation**).
  - Normalization breaks  $R$  into smaller relations.
  - $R$  could have been the result of some ad hoc design of relations, which we then test/convert to normal form.

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## ER Model and Normalization

- When an E-R diagram is carefully designed, identifying all entities correctly, the tables generated from the E-R diagram should not need further normalization.
- However, in a real (imperfect) design, there can be FDs from non-key attributes of an entity to other attributes of the entity
  - Example: an *employee* entity with attributes *department\_number* and *department\_address*, and a functional dependency  $department\_number \rightarrow department\_address$
  - Good design would have made department an entity
- Functional dependencies from non-key attributes of a relationship set possible, but rare --- most relationships are binary

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## Denormalization for Performance

- May want to use non-normalized schema for performance
- Eg, displaying *customer\_name* along with *account\_number* and *balance* requires join of *account* with *depositor*
- Alternative 1: Use **denormalized relation** containing all above attributes
  - faster lookup
  - extra space and extra execution time for updates
  - extra coding work and possibility of error in extra code
- Alternative 2: use a **materialized view** defined as  $account \bowtie depositor$ 
  - Benefits and drawbacks same as above, except no extra coding work for programmer and avoids possible errors

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## Other Design Issues

- Some aspects of database design are not caught by normalization
- Examples of bad database design, to be avoided:  
Instead of *earnings(company\_id, year, amount)*, use *earnings\_2013, earnings\_2014, earnings\_2015, etc.*, all on the schema (*company\_id, earnings*).
- Above are in BCNF, but make querying across years difficult and needs new table each year

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## Other Design Issues (cont.)

- *company\_year(company\_id, earnings\_2013, earnings\_2014, earnings\_2015)*
  - Also in BCNF, but also makes querying across years difficult and requires new attribute each year.
  - Is an example of a **crosstab**, where values for one attribute become column names
  - Used in spreadsheets, and in data analysis tools

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## Recap

- Designing a Set of Relations
- Properties of Relational Decompositions
- Algorithms for Relational Database Schema
- Multivalued Dependencies and Fourth Normal Form
- Join Dependencies and Fifth Normal Form
- Other Dependencies and Normal Forms

## Assignment 5

- Textbook exercises: 7.21, 7.22, 7.29, 7.30, 7.31
- Due date: **Jan 13, 2021**