# CSIEB0100 Data Structures

## Lecture 04 Stacks and Queus

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Lecture material is partly home-grown, partly taken from slides came with the textbook originally prepared by Professor Jiun-Long Huang of NCTU.

## Templates in C++

- Templates in C++ makes it easier to reuse classes and functions.
- A template can be viewed as a variable that can be instantiated to any data type, irrespective of whether this data type is a fundamental C++ type or a userdefined type.
- C++ support class templates and function templates.
- Class templates are well suited for implementing ADTs.

## Selection Sort Using Template

```
Template <class KeyType>
void sort(KeyType *a, int n)
// sort the n KeyTypes a[0] to a[n-1] into nondecreasing order
{
  for (int i = 0; i < n; i++)
  {
    int j = i;
    // find smallest KeyType in a[i] to a[n-1]
    for (int k = i+1; k < n; k++)
      if (a[k] < a[j]) { j = k; }
    // interchange
    KeyType temp = a[i];
    a[i] = a[j];
    a[j] = temp;
  }
}
```

Can sort array of values in any KeyType

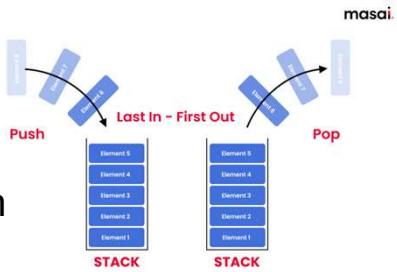


## Selection Sort Using Template

```
float farray[100];
int intarray[250];
……
// Assume that the arrays are initialized
// at this point
sort(farray, 100);
sort(intarray, 250);
```

# Stack

- Last-In-First-Out (LIFO) list
- Push (Add) : Add an element into the stack.
- Pop (Delete) : Get and delete the top element from the stack.

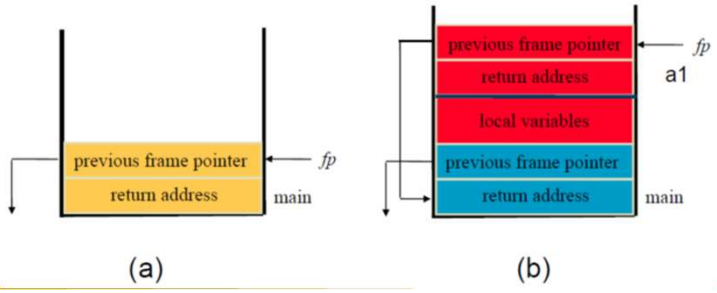



- Peek : Retrieve the top element **without** deleting it.
- IsEmpty : Check if the stack is empty or not
- IsFull : Check if the stack is full or not



# Stack Example: Call Stack

- A call stack is used to keep information about active function calls and to guide the execution.
- When a function is called, a stack frame with the right return address (the address after the call instruction) is pushed on top of the call stack.



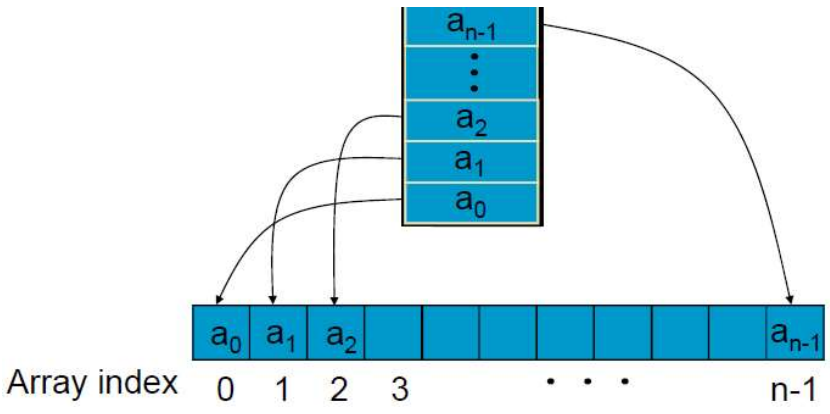
# The Stack Template

```
Template <class KeyType>
class Stack
{  // objects: A finite ordered list with zero or more elements
 public:
   Stack (int MaxStackSize = DefaultSize);
  // Create an empty stack whose maximum size is MaxStackSize
   Boolean IsFull();
  // if number of elements in the stack is equal to the maximum size
  // of the stack, return TRUE(1) else return FALSE(0)
   void Add(const KeyType& item);
  // if IsFull(), then StackFull(); else insert item into the top of the
stack.
   Boolean IsEmpty();
  // if number of elements in the stack is 0, return TRUE(1) else
  //return FALSE(0)
   KeyType* Delete(KeyType& );
  // if IsEmpty(), then StackEmpty() and return 0;
  // else remove and return a pointer to the top element of the stack.
};
```



# Implementation of Stack by Array

- How to check whether a stack is full or empty?



```
...
  void StackEmpty() {cout << "empty" << endl;};
  void StackFull() {cout << "full" << endl;};
  void Output();
Private:
  int top;
  KeyType *stack;
  int MaxSize;
...
template<class KeyType>
Stack<KeyType>::Stack(int MaxStackSize):
MaxSize(MaxStackSize) {
  stack=new KeyType[MaxSize];
  top=-1;
}
```

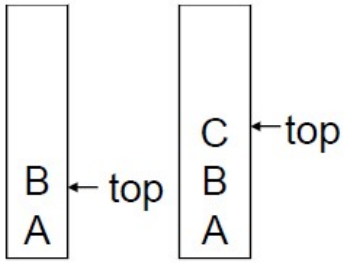


```
template<classs KeyType>
inline Boolean Stack<KeyType>::IsFull() {
  if (top==MaxSize-1) return TRUE;
  else return FALSE;
}

template<classs KeyType>
inline Boolean Stack<KeyType>::IsEmpty() {
  if (top==-1) return TRUE;
  else return FALSE;
}
```

## Add an Item to the Stack

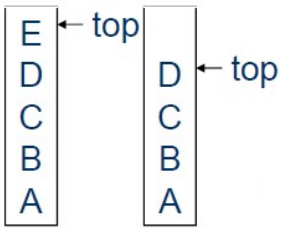
```
Template <class KeyType>
void Stack<KeyType>::Add(const KeyType& x)
{
  /* add an item to the global stack */
  if ( IsFull() )
    StackFull();
  else
    stack[++top]=x;
}
```





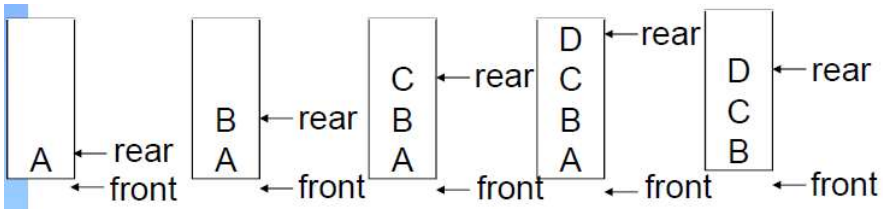

## Deleting from the Stack

```
Template <class KeyType>
KeyType* Stack<KeyType>::Delete(KeyType& x)
{
  // return the top element from the stack
  if ( IsEmpty() )
  {
    StackEmpty();
    /* returns error status */
    return 0;
  }
  x=stack[top--];
  return &x;
}
```



## Queue: First-In-First-Out(FIFO) List

- Add an element into a queue (at rear)
- Get and delete an element from a queue (at front)
- Variation
  - Priority queue






## Application: Job Scheduling

| front | rear | Q[0] | Q[1] | Q[2] | Q[3] | Comments |
|---|---|---|---|---|---|---|
| -1 | -1 | | | | | Queue is empty |
| -1 | 0 | J1 | | | | J1 is added |
| -1 | 1 | J1 | J2 | | | J2 is added |
| -1 | 2 | J1 | J2 | J3 | | J3 is added |
| 0 | 2 | | J2 | J3 | | J1 is deleted |
| 1 | 2 | | | J3 | | J2 is deleted |

- Insertion and deletion from a sequential queue

# The Queue Template

```
template <class KeyType>
class Queue
{
// objects: A finite ordered list with zero or more elements.
public:
   Queue (int MaxQueueSize = DefaultSize);
   // Create an empty queue whose maximum size is \fIMaxQueueSize\fR

   Boolean IsFull ();
   // if number of elements in the queue is equal to the maximum size of
   //   the queue, return TRUE (1); otherwise, return FALSE (0)

   void Add (const KeyType& item);
   // if IsFull(), then QueueFull(); else insert item into the top of the queue.

   Boolean IsEmpty ();
   // if number of elements in the queue is equal to 0, return TRUE (1) else return FALSE (0).

   KeyType* Delete (KeyType&);
   // if IsEmpty(), then QueueEmpty(); else remove and return the topmost element of the Queue

   void QueueEmpty() {cout << "empty" << endl;};
   void QueueFull() {cout << "full" << endl;};
```



# Implementation 1: Using Array

```
private:
    int front;
    int rear;
    KeyType *queue;
    int MaxSize;
};

template <class KeyType>
Queue<KeyType>::Queue (int MaxQueueSize) :
  MaxSize(MaxQueueSize)
{
    queue = new KeyType[MaxSize];
    front = rear = -1;
}
```

```
template <class KeyType>
inline Boolean  Queue<KeyType>::IsFull()
{
    if (rear == MaxSize-1) return TRUE;
    else return FALSE;
}

template <class KeyType>
inline Boolean  Queue<KeyType>::IsEmpty()
{
    if (front == rear) return TRUE;
    else return FALSE;
}
```



## Add to a Queue

```
template <class KeyType>
void Queue<KeyType>::Add(const KeyType& x)
// add x to the queue
{
    if ( IsFull() )
        QueueFull();
    else
        queue[++rear] = x;
}
```

## Delete an Element from Queue

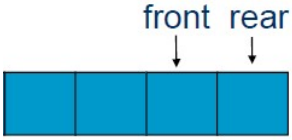
```
template <class KeyType>
KeyType* Queue<KeyType>::Delete (KeyType& x)
// remove and return front element from queue
{
  if ( IsEmpty() ) {
    QueueEmpty();
    return 0;
  }
  x = queue[++front];
  return &x;
}
```

Problem:
There may be available space when QueueFull is true i.e. data movements are required.



## Problem with the Implementation

- As the elements enter and leave the queue, the queue gradually shifts to the right.
  - Eventually the rear index equals *MaxSize-1,* suggesting that the queue is full even though the underlying array is not full
- Solution:
  - Use a function to move the entire queue to the left so that *front=-1*
  - It is time-consuming
  - Time complexity=O(MaxSize)

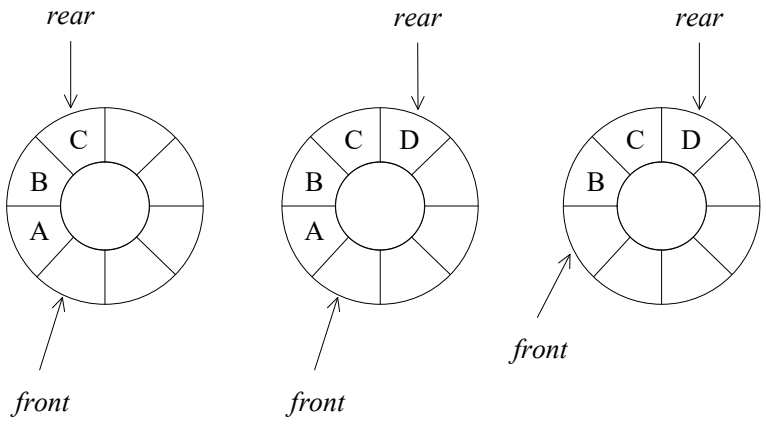


## Implementation 2: Regard an Array as a Circular Queue

- Two indices
  - **front**: one position counterclockwise from the first element
  - **rear**: current end
- Problem
  - In order to distinguish whether a circular queue is full or empty, one space is left when queue is full

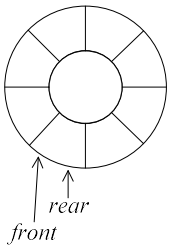


## An Example Circular Queue

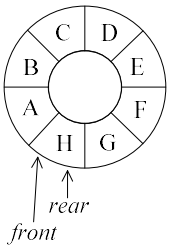



(a) Initial (b) Add (c) Delete

## An Empty and a Full Queue




(a) 圖3.8(a)的佇列連續執行了三次的移除動作後的結果

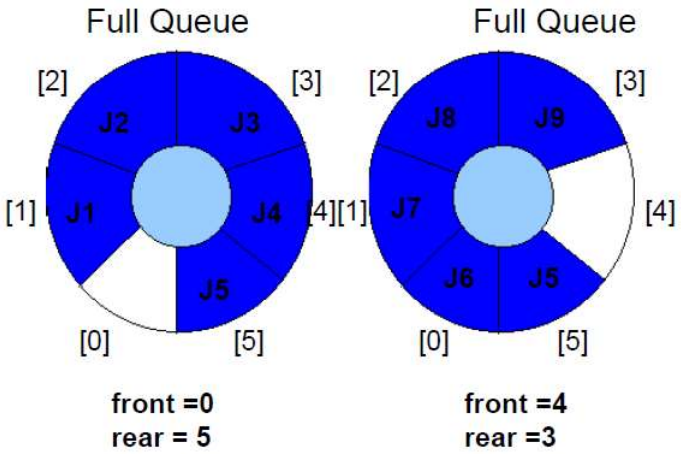



(b) 圖3.8(a)的佇列連續做了五次的加入動作後的結果

- When front=rear, cannot distinguish between an empty and a full queue.



## Full Queues




Full circular queues and then we remove the item

## Add to a Circular Queue

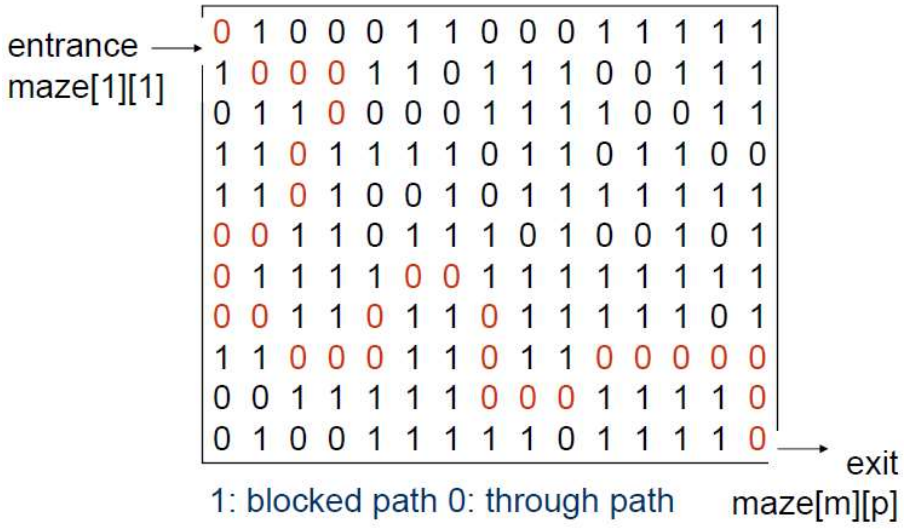
```
Template<class KeyType>
void Queue<KeyType>::Add(const KeyType& x)
{
  int newrear = (rear+1) % MaxSize;
  if ( front==newrear )
    QueueFull();
  else
    queue[rear=newrear]=x;
}
```



## Deleting from a Circular Queue

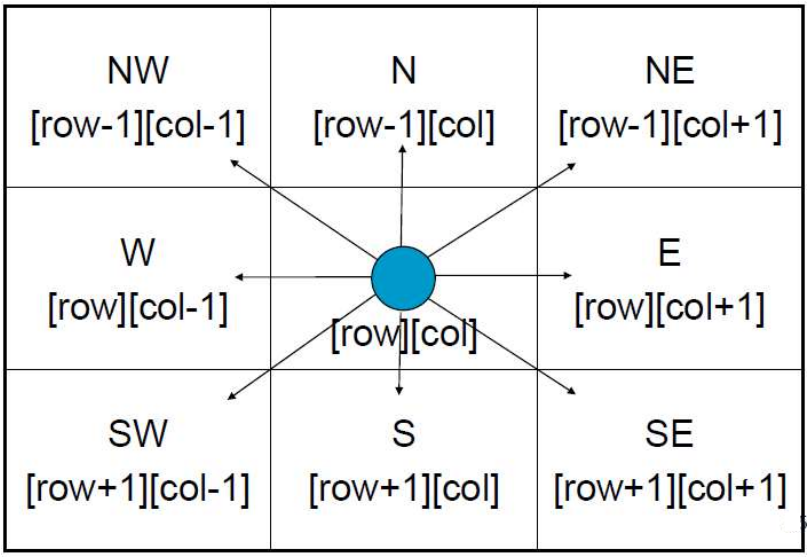
```
Template<class KeyType>
KeyType* Queue<KeyType>::Delete(KeyType& x)
{
  /* remove front element from the queue */
  if (front == rear)
  {
    QueueEmpty();
    return 0;
  }
  front = (front+1) % MaxSize;
  x=queue[front];
  return &x;
}
```

# A Mazing Problem






# A Possible Representation



```
typedef struct {
  int a; /* row */
  int b; /* col */
} offsets;
offsets move[8];
/*array of moves for each direction*/
```

```
next_row = row + move[dir].a;
next_col  = col  +  move[dir].b;
```

| Name | Dir | Move[dir].a | Move[dir].b |
|---|---|---|---|
| N | 0 | -1 | 0 |
| NE | 1 | -1 | 1 |
| E | 2 | 0 | 1 |
| SE | 3 | 1 | 1 |
| S | 4 | 1 | 0 |
| SW | 5 | 1 | -1 |
| W | 6 | 0 | -1 |
| NW | 7 | -1 | -1 |



# Use a Stack to Keep Past History

- What is the maximal size of the stack?
  - A maze is represented by a two dimensional array *maze[m][p]*
  - Since each position is visited at most once, $m \times p$ elements(at most) can be placed in the stack

```
typedef struct {
  int x;
  int y;
  int dir;
} item;
item stack[m*p];
```

```
→ 0 0 0 0                          (2,4,6) (2,3) Mark (2,3)
  0 1 0 0                          (1,4,4)
  0 1 1 1                          (1,3,2)
  0 0 0 0 →                        (1,2,2)
                                   (1,1,2)
Stack          Action
      Position
------- (1,1) Mark (1,1)   Move back

(1,1,2) (1,2) Mark (1,2)   (1,4,4) (2,4)
                           (1,3,2)
(1,2,2) (1,3) Mark (1,3)   (1,2,2)
(1,1,2)                    (1,1,2)

(1,3,2) (1,4) Mark (1,4)   Move back
(1,2,2)
(1,1,2)                    (1,3,2) (1,4)
                           (1,2,2)
(1,4,4) (2,4) Mark (2,4)   (1,1,2)
(1,3,2)
(1,2,2)                    ...
(1,1,2)
```



```
initialize stack to the maze entrance coordinates and direction
   east ;
while (stack is not empty)
{
  (i, j, dir) = coordinates and direction deleted from top of
   stack ;
  while (there are more moves) // try all possible moves
  {
    (g, h) = coordinates of next move ;
    if ((g == m) && (h == p)) success ;
    if ((!maze[g][h]) // legal move
        && (!mark[g][h]) // haven't been here before
    {
      mark[g][h] = 1 ;
      dir = next direction to try if we backtrack to (i, j);
      add (i, j, dir) to top of stack ;
      i = g ; j = h ; dir = north ; // starts with north
    }
  }
}
cout << "no path found" << endl ;
```

```
void path(int m, int p)
/* Output a path (if any) in the maze;
   maze[0][i]=maze[m+1][i]= maze[j][0]=maze[j][p+1]=1,
   0<=i<=p+1, 0<=j<=m+1. */
{
  //start at (1,1)
  mark[1][1] = 1;
  Stack<items> stack(m*p);
  items temp;
  temp.x = 1; temp.y = 1; temp.dir = E;
  stack.Add(temp);

  while (!stack.IsEmpty()) // stack not empty
  {
    temp = *stack.Delete(temp); // unstack
    int i = temp.x; int j = temp.y; int d = temp.dir;
    while (d < 8) // move forward
    {
      int g = i+move[d].a; int h = j+move[d].b;
```


(m+2)*(p+2)
To build borders



```
      if ((g == m) && (h == p)) { // reached exit
        // output path
        cout << stack;
        cout << i << " " << j << endl; // last two squares
        cout << m << " " << p << endl;
        return;
      }
      if ((!maze[g][h]) && (!mark[g][h])) { // new pos
        mark[g][h] = 1;
        temp.x = i;  temp.y = j;  temp.dir = d+1;
        stack.Add(temp); // stack it
        i = g; j = h; d = N; // move to (g,h)
      }
      else d++; // try next direction
    }
  }
  cout << "no path in maze " << endl;
}
```

## Evaluation of Expressions

- X = a / b - c + d * e - a * c
- a = 4, b = c = 2, d = e = 3, What is the value of X?
- Interpretation 1: *, / have higher precedence
  - ((4/2)-2)+(3*3)-(4*2)=0 + 9 - 8=1
- Interpretation 2: +, - have higher precedence
  - (4/(2-2+3))*(3-4)*2=(4/3)*(-1)*2= -2.66666…
- How to generate the machine instructions corresponding to a given expression?
  - Precedence rule + associative rule



## Notations of Expressions

- Infix:
  - Each operator comes in-between the operands
  - 2+3
- Postfix
  - Each operator appears after its operands
  - 23+
- Prefix
  - Each operator appears before its operands
  - +23

# For User vs. Computer

| user | computer |
|---|---|
| Infix | Postfix |
| 2+3*4 | 234*+ |
| a*b+5 | ab*5+ |
| (1+2)*7 | 12+7* |
| a*b/c | ab*c/ |
| (a/(b-c+d))*(e-a)*c | abc-d+/ea-*c* |
| a/b-c+d*e-a*c | ab/c-de*ac*-+ |

- Postfix & prefix: no parentheses, no precedence



# Evaluation of Expressions

- Phase 1: Infix to postfix conversion
  - 6/2-3+4*2 → 6 2 / 3 – 4 2 * +
- Phase 2: Postfix expression evaluation
  - 6 2 / 3 – 4 2 * + → 8

# Phase 2: Postfix Expression Evaluation using Stack

- 6 2 / 3 – 4 2 * +

| Token | Stack [0] | [1] | [2] | Top |
|---|---|---|---|---|
| 6 | 6 | | | 0 |
| 2 | 6 | 2 | | 1 |
| / | 3 | | | 0 |
| 3 | 3 | 3 | | 1 |
| - | 0 | | | 0 |
| 4 | 0 | 4 | | 1 |
| 2 | 0 | 4 | 2 | 2 |
| * | 0 | 8 | | 1 |
| + | 8 | | | 0 |



Program 3.18

```
void eval(expression e)
/* Evaluate the postfix expression e. It is assumed
that the last token ( a token is either an operator,
operand, or '#') in e is '#'. A function NextToken
is used to get the next token from e. The function
uses the stack stack */
{
   Stack<token> stack ; //initialize stack
   for(token x=NextToken(e);x!='#';x=NextToken(e))
      if(x is an operand) stack.Add(x) // add to
stack
      else
      { //operator
        remove the correct number of operands for
operator x from stack ;
        perform the operation x and store the
result (if any) onto the stack ;
      }
} // end of eval
```

# Phase 1: Infix to Postfix Conversion

- Assumptions:
  - operators: +, -, *, /, %
  - operands: single digit integer



# Intuitive Algorithm

(1) Fully parenthesize expression

a / b - c + d * e - a * c -->

((((a / b) - c) + (d * e)) – (a * c))

(2) All operators replace their corresponding right parentheses.

((((a / b) - c) + (d * e)) - a * c))


(3) Delete all parentheses.

ab/c-de*+ac*-

Two passes

## An Observation

- The orders of operands in infix and postfix are the same.

a + b * c, * > +

| Token | Stack [0] [1] [2] | Top | Output |
|---|---|---|---|
| a | | -1 | a |
| + | + | 0 | a |
| b | + | 0 | ab |
| * | + * | 1 | ab |
| c | + * | 1 | abc |
| <eos> | | -1 | abc*+ |



## Another Example

a * b + c, * > +

| Token | Stack [0] [1] [2] | Top | Output |
|---|---|---|---|
| a | | -1 | a |
| * | * | 0 | a |
| b | * | 0 | ab |
| + | + | 1 | ab* |
| c | + | 1 | ab*c |
| <eos> | | -1 | ab*c+ |

# Yet Another Example

a $*_1$ (b + c) $*_2$ d

| Token | Stack [0] [1] [2] | Top | Output |
|---|---|---|---|
| a | | -1 | a |
| $*_1$ | $*_1$ | 0 | a |
| ( | $*_1$ ( | 1 | a |
| b | $*_1$ ( | 1 | ab |
| + | $*_1$ ( + | 2 | ab |
| c | $*_1$ ( + | 2 | abc |
| ) | $*_1$ match ( | 0 | abc+ |
| $*_2$ | $*_2$ $*_1 = *_2$ | 0 | abc+$*_1$ |
| d | $*_2$ | 0 | abc+$*_1$d |
| <eos> | | -1 | abc+$*_1$d$*_2$ |



# Rules and Complexity

- Operators are taken out of the stack if their in-stack priority is numerically less than or equal to (equal or higher precedence) the in-coming priority of the new operator, i.e., isp(y)<=icp(x)
- Assume that isp("(")=8 and icp("(")=0
  - No operator other than the matching right parenthesis ")" should cause it to get unstacked
  - On ")", keeps poping until the matching "("
- isp('#')=icp('#')=8
- Complexity: O(n)

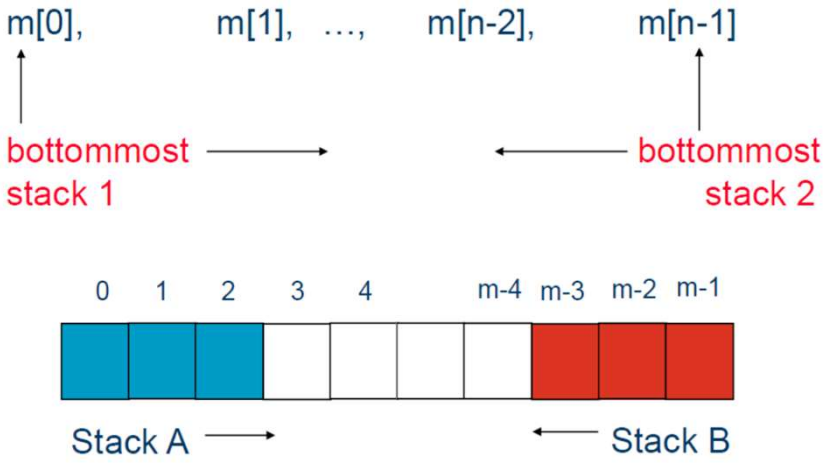
# Operator Priority

| Priority | Operator |
|---|---|
| 1 | Unary minus, ! |
| 2 | *,/,% |
| 3 | +,- |
| 4 | <,<=,>=,> |
| 5 | ==,!= |
| 6 | && |
| 7 | \|\| |



# Multiple Stacks and Queues

- Two stacks



# Multiple Stacks and Queues

- More than two stacks or queues (n)
- Memory is divided into n segments
  - The initial division of these segments may be done in proportion to expected sizes of these stacks or queues if these are known
  - All stacks or queues are empty and divided into roughly equal segments



# Multiple Stacks and Queues

- boundary[stack_no]
  - 0 ≤ stack_no < MAX_STACKS
- top[stack_no]
  - 0 ≤ stack_no < MAX_STACKS