A Dynamic Calculation Method for AHP in Unexpected Environments

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Abstract

AHP (Analytic Hierarchy Process) method is a systematic method that provides a comprehensive structure to combine the intuitive rational and irrational values during the decision-making process by a pair-wise comparison approach for comparing a list of objectives or alternatives. However, the AHP process is time-consuming and the decision model is static. It cannot reflect the change brought by a sudden incident to the relative weights of decision criteria. In this paper, we present an algorithm that can determine the priorities of alternatives from adjusted relative weights. In other words, if the relative weights of the prime criteria are changed, how will the ranking be affected? Our algorithm can deal with the change by the decision maker's preferences have changed. And it helps decision-maker(s) to make more effective decisions.

Keywords: AHP Decision, Dynamic Model, Unexpected Accident, Pair-wise Comparison, Static Model

1. Introduction

AHP is one of the major techniques in dealing with MCDM problem that was originally developed by Professor Thomas L. Saaty (1980, 1990, 2003, and 2008). It is based upon making pair-wise comparisons between the decision alternatives for each criterion, enabling the ranking of the decision alternatives to be achieved. It is helpful for decision makers to structure the problems, conduct analysis, and rank the alternatives (Jiang et al. 2011). Due to its logicality, rationality, and computational simplicity, AHP has been widely applied to the research of evaluation and selection problems and risk analysis problems.

AHP is based on seven steps, and depicted below (Vaidya et al, 2006;Wu et al. 2012): (1) State the problem; (2) Broaden the objectives of the problem by considering all actors, objectives, and outcomes; (3) Identify the criteria and/or sub-criteria; (4) Structure the problem hierarchically by considering the goal, criteria, sub-criteria, and a set of alternatives; (5) Construct a set of pair-wise comparison matrices; (6) Perform computations to find the maximum eigen-value, consistency index, consistency ratio (CR), and normalized values for criteria and/or sub-criteria and alternative; and (7) Use the normalized values to make decisions if CR is satisfactory with the value less than 0.1. Furthermore, AHP allow some small inconsistency in judgment because human is not always consistent.

Additionally, because of its flexibility, it can be integrated with other methods, e.g., QFD (Quality Function Deployment) (Lu et al. 1994; Bhattacharya et al., 2005; Vaidya et al, 2006; Rajesh et al. 2013), DEA (Data Envelopment Analysis) (Liu et al., 2005; Zhang et al., 2006), meta-heuristics (Rad et al., 2008), and SWOT (Strengths-Weaknesses-Opportunities-Threats) (Kurttila Mikko et al., 2000; Ram et al., 2004;), etc. (Ho 2007; Diamantopoulos 2012; Liu et al. 2013) This enables the user to extract benefits from all the combined methods, and hence, achieve the desired goal in a better way (Vaidya et al. 2006).

The AHP also allows group decision making, where group members can use their experience, values and knowledge to break down a problem into a decision hierarchy and solve it by the AHP steps (Kamal M. Al-Subhi et al. 2001; Liu et al., 2013). The application of the AHP to support group decisions has proven to be contributive in several research studies. According to Dyer and Forman (1992) also believe that the AHP is well suited to group decision making and that it can be applied to a variety of group decision contexts. They argued that the AHP can help group decision makers' structure complex decisions, develop measures of utility, and synthesize measures of both tangibles and intangibles with respect to the numerous competing objectives inherent in almost any decision (Lai et al. 2002).

Although AHP which is known as a powerful decision-making process to help decision maker(s) make the best possible decisions. But AHP is a static model. It can't deal with the change by the decision maker's preferences have changed. And humans have a natural cognitive bias towards giving too much weight to unusual events. For example, by February 2013, the Abenomics policy led to a dramatic weakening of the Japanese yen. If the decision-maker(s)'s preferences change over time, the matrix will re-calculate.

The purpose of this paper is to provide an algorithm that can determine the priorities of alternatives from adjusted relative weights.

2. Literature Review

Decision-making are one of the most central processes in organizations and a basic task of management at all levels. According to Cole (2004), decision-making is a process of identifying a problem, evaluating alternatives, and selecting one alternative. The AHP is a powerful and flexible decision-making process (Saaty, 1980) to help managers set priorities and make the best decision when both qualitative and quantitative aspects of a decision need to be considered. The benefits of the AHP include its ability to handle multiple stakeholders with multiple objectives, the inclusion of possible interaction effects and the relative ease of computation (Weiss et al., 1987).

However, in real-world application, the human preference model is uncertain and decision-makers may be unable to assign exact numerical values to the comparison judgments. For instance, when evaluating different suppliers, the decision-makers are usually unsure about their level of preference due to incomplete and uncertain information about possible suppliers and their performances. Since some of the supplier evaluation criteria are subjective and qualitative, it is very difficult for the decision-maker to express the strength of his preferences and to provide exact pair-wise comparison judgments. For this reason, we need a more effective method than the original AHP that can help us to make more accurate decisions (Bellman and Zadeh, 1970).

Today many researchers and practitioners are working in dynamic decision-making related research topics. According to Benítez et al. (2012) propose a framework that allows users to provide partial and/or incomplete preference data at multiple times. According to Chiang (2005) proposed a dynamic decision approach for long-term vendor selection based on AHP and BSC (Balanced Score-Card) for purpose of choosing the sellers. According to Searcy (2004) also suggests integrating AHP and BSC for estimating the performance of enterprises to structure the analytic frameworks. According to Lin et al. (2008) proposes an adaptive AHP approach (A^3) that uses a soft computing scheme, Genetic Algorithms, to recover the real number weightings of the various criteria in AHP and provides a function for automatically improving the consistency ratio of pairwise comparisons. According to Duleba et al. (2012) propose an algorithm for scoring so that the missing data of the matrices could be calculated.

Dynamic analytic hierarchy process is the method considering the factor of time in AHP model, and the judgment matrices are time dependent functions, named dynamic judgment matrices (Saaty, 1980; Li, 1997; Gao et al. 2011). Saaty (1980) gives several normal functions in dynamic judgment matrices and discusses the corresponding solutions, but to find the analytical solution is very difficult. So far, a few methods have been proposed for solving this problem, including least perturbations method (Xu, 2004), least square method (Jensen, 1984), and goal programming method (Bryson, 1995). Saaty (2007) expressed There are essentially two analytic ways to study dynamic decisions: structural, by including scenarios and time periods as elements in the structure that represents a decision, and functional by explicitly involving time in the judgment process. A possible third way would be a hybrid of these two.

3. Methodology

We first provide AHP method to help an iron and steel firm's decision makers facing a complex problem with multiple-criteria to evaluate and select the best supplier. Then define the different tasks in dynamic decision-making under uncertainty as problem analysis and problem solution. But the results indicate that AHP is a systematic analysis methodology, nevertheless it can't allow decision maker(s) to adjust the criteria and/or sub-criteria promptly. In order to improve the AHP architect cannot display the change brought by an unexpected incident. And it cannot show accurately the dependency among criteria.

Following are the mathematical that are sensitivity analysis investigates how our decision might change given a change in two criteria.

Case 1. $k < p$

(1) Priority Vector's Numerator

$$\left(a_{k1}a_{k2}\ldots\ldots a_{kn}\right)^{\frac{1}{n}}=\left(\sum_{j=1}^{n}a_{kj}\right)^{\frac{1}{n}}\ldots\ldots\ldots\ldots\ldots\ldots(E_k)$$

(2) Priority Vector

$$w_k=\frac{\left(\prod_{j=1}^{n}a_{kj}\right)^{\frac{1}{n}}}{\left(\prod_{j=1}^{n}a_{pj}\right)^{\frac{1}{n}}+\left(\prod_{j=1}^{n}a_{qj}\right)^{\frac{1}{n}}+\sum_{i\in\{1\sim p-1,\,p+1\sim q-1,\,q+1\sim n\}}^{n}\left(\prod_{j=1}^{n}a_{ij}\right)^{\frac{1}{n}}}=\frac{\left(E_k\right)^{\frac{1}{n}}}{\sum_{i}^{n}\left(E_i\right)^{\frac{1}{n}}}$$

Case 2. $k = p$

(1) Priority Vector's Numerator

$$\left(a_{p1}a_{p2}\ldots\ldots a_{p(q-1)}\left(a_{pq}^{new}\right)a_{p(q+1)}\ldots\ldots.a_{pn}\right)^{\frac{1}{n}}$$
$$=\left(\sum_{j=1}^{q-1}a_{ij}\left(a_{pq}+\alpha\right)\sum_{j=q+1}^{n}a_{ij}\right)\ldots\ldots\ldots\ldots\ldots\ldots(E_p)$$

(2) Priority Vector

$$w_p=\frac{\left(\prod_{j=1}^{n}a_{pj}\right)^{\frac{1}{n}}}{\left(\prod_{j=1}^{n}a_{pj}\right)^{\frac{1}{n}}+\left(\prod_{j=1}^{n}a_{qj}\right)^{\frac{1}{n}}+\sum_{i\in\{1\sim p-1,\,p+1\sim q-1,\,q+1\sim n\}}^{n}\left(\prod_{j=1}^{n}a_{ij}\right)^{\frac{1}{n}}}=\frac{\left(E_p\right)^{\frac{1}{n}}}{\sum_{i}^{n}\left(E_i\right)^{\frac{1}{n}}}$$

$Case\ 3.\ p < k < p$

(1) Priority Vector's Numerator

$$(a_{k1}a_{k2}\dots\dots a_{kn})^{\frac{1}{n}} = \left(\sum_{j=1}^{n} a_{kj}\right)^{\frac{1}{n}} \dots\dots\dots\dots\dots\dots (E_k)$$

(2) Priority Vector

$$w_k = \frac{\left(\prod_{j=1}^{n} a_{kj}\right)^{\frac{1}{n}}}{\left(\prod_{j=1}^{n} a_{pj}\right)^{\frac{1}{n}} + \left(\prod_{j=1}^{n} a_{qj}\right)^{\frac{1}{n}} + \sum_{i\in\{1\sim p-1,\, p+1\sim q-1,\, q+1\sim n\}}^{n} \left(\prod_{j=1}^{n} a_{ij}\right)^{\frac{1}{n}}} = \frac{(E_k)^{\frac{1}{n}}}{\sum_{i}^{n} (E_i)^{\frac{1}{n}}}$$

$Case\ 4.\ k = q$

(1) Priority Vector's Numerator

$$\left(a_{(p-1)1}a_{(p-2)2}\dots\dots a_{(p-1)(q-1)}\left(a_{qp}^{new}\right)a_{(p-1)(q+1)}\dots\dots .a_{(p-1)n}\right)^{\frac{1}{n}}$$

$$= \left(a_{(p-1)1}a_{(p-2)2}\dots\dots a_{(p-1)(q-1)}\left(\frac{1}{a_{pq}+\alpha}\right)a_{(p-1)(q+1)}\dots\dots .a_{(p-1)n}\right)^{\frac{1}{n}}$$

$$= \left(\sum_{j=1}^{p-1} a_{ij}\left(\frac{1}{a_{pq}+\alpha}\right)\sum_{j=q+1}^{n} a_{ij}\right) \dots\dots\dots\dots\dots\dots (E_q)$$

(2) Priority Vector

$$w_p = \frac{\left(\prod_{j=1}^{n} a_{qj}\right)^{\frac{1}{n}}}{\left(\prod_{j=1}^{n} a_{pj}\right)^{\frac{1}{n}} + \left(\prod_{j=1}^{n} a_{qj}\right)^{\frac{1}{n}} + \sum_{i\in\{1\sim p-1,\, p+1\sim q-1,\, q+1\sim n\}}^{n} \left(\prod_{j=1}^{n} a_{ij}\right)^{\frac{1}{n}}} = \frac{(E_q)^{\frac{1}{n}}}{\sum_{i}^{n} (E_i)^{\frac{1}{n}}}$$

$Case\ 5.\ k > q$

(1) Priority Vector's Numerator

$$(a_{k1}a_{k2}\dots\dots a_{kn})^{\frac{1}{n}} = \left(\sum_{j=1}^{n} a_{kj}\right)^{\frac{1}{n}} \dots\dots\dots\dots\dots\dots (E_k)$$

(2) Priority Vector

$$w_k = \frac{\left(\prod_{j=1}^{n} a_{kj}\right)^{\frac{1}{n}}}{\left(\prod_{j=1}^{n} a_{pj}\right)^{\frac{1}{n}} + \left(\prod_{j=1}^{n} a_{qj}\right)^{\frac{1}{n}} + \sum_{i\in\{1\sim p-1,\, p+1\sim q-1,\, q+1\sim n\}}^{n} \left(\prod_{j=1}^{n} a_{ij}\right)^{\frac{1}{n}}} = \frac{(E_k)^{\frac{1}{n}}}{\sum_{i}^{n} (E_i)^{\frac{1}{n}}}$$

Before we start to solve the problem, we first assume the following terms:

1. $(a_{pq}^{new} = a_{pq} + \alpha,\ a_{qp}^{new} = \frac{1}{a_{pq} + \alpha})$

2. $a_{pq} + \alpha > 0$

3. other elements's relative wights unchanged.

4. pair-wise matrixes remain consistent.

Following are the examples. In Table 1, we can see if we change the prefers (Criterion D $\succ$ Criterion C) then the value changed (2 $\succ$ 1/5). The results of this analysis are displayed in the Table 2.

Table 1 Original Matrix:

	Criterion A	Criterion B	Criterion C	Criterion D	Weight	Ranking
Criterion A	1	1	3	5	1.968	1
Criterion B	1	1	2	4	1.682	2
Criterion C	$\frac{1}{3}$	$\frac{1}{2}$	1	5	0.955	3
Criterion D	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{5}$	1	0.316	4

Table 2 New Matrix:

	Criterion A	Criterion B	Criterion C	Criterion D	Weight	Ranking
Criterion A	1	1	3	5	1.968	1
Criterion B	1	1	2	4	1.682	2
Criterion C	$\frac{1}{3}$	$\frac{1}{2}$	1	$\frac{1}{2}$	0.537	4
Criterion D	$\frac{1}{5}$	$\frac{1}{4}$	2	1	0.562	3

$$(1)\forall k, \text{if } \mathrm{R}(k), \mathrm{R}(k+1) \neq \mathrm{p, q}$$

$$(\prod_{j=1}^{n} a_{R(2)j}^{new})^{\frac{1}{n}} = (1\times1\times2\times4)^{\frac{1}{4}} = 1.682$$

$$(\prod_{j=1}^{n} a_{R(1)j}^{new})^{\frac{1}{n}} = (1\times1\times3\times5)^{\frac{1}{4}} = 1.968$$

$$\because (\prod_{j=1}^{n} a_{R(k+1)j}^{new})^{\frac{1}{n}} < (\prod_{j=1}^{n} a_{R(k)j}^{new})^{\frac{1}{n}}$$

$$\Rightarrow w_{R(2)}^{new} < w_{R(1)}^{new}$$

$$(2) R(k) \neq q, R(k+1) = p$$

$$\alpha > a_{pq}((\frac{\omega_{R(k)}}{\omega_{R(k+1)}})^{n} - 1)$$

$$\Rightarrow \alpha > 2\times((\frac{1.968}{1.682})^{4} - 1)$$

$$\Rightarrow \alpha > 1.75$$

If $\alpha > 1.75$, then reverse the sequence of Supplier A and Supplier B.

4. Conclusions

As mentioned previously, we have proposed an efficient algorithm to identify changes in pair-wise comparison matrix that may be triggered by an unexpected incident. In the future, we plan to link AHP and ECA rule base to develop a tool based on this calculation method. We expect that this calculation method will become practical in the iron and steel industry.

REFERENCES

Bellman, R., and Zadeh, L. A. 1970. Decision Making in a Fuzzy Environment. Management Science, 17(4), 141–164.

Bryson, N. 1995. A Goal Programming Method for Generating Priority Vectors, Journal of Operational Research, 46, 641–648.

Bhattacharya, A., Sarkar, B., and Mukherjee S. K. 2005. Integrating AHP with QFD for Robot Selection under Requirement Perspective, International Journal of Production Research, 43(17), 3671-3685.

Benítez, J., Delgado-Galván, X., Izquierdo, J., Pérez-García, R. 2012. An Approach to AHP Decision in a Dynamic Context, Journal of Decision Support Systems, 53(3), 499-506.

Cole, G. A., 2004. Management Theory and Practice, Publisher: Thomson Learning

EMEA, London, United Kingdom.
Cleotilde Gonzalez, 2005. Decision Support for Real-Time, Dynamic Decision-Making Tasks, Organizational Behavior and Human Decision Processes, 96, 142-154.
Chiang, Z. 2005. A Dynamic Decision Approach for Long-Term Vendor Selection Based on AHP and BSC, Journal of Advances in Intelligent Computing, 3645, 2005, 257-265.
Dyer, R.F., and Forman, E. H. 1992. Group Decision Support with the Analytic Hierarch Process, Decision Support Systems, 8, 99-124.
Diamantopoulos, A., Fritz, W., and Hildebrandt, L. 2012. Quantitative Marketing and Marketing Management: Marketing Models in Theory and Practice, Springer Gabler Wiesbaden.
Duleba, S.; Mishina, T.; Shimazaki, Y. 2012. A Dynamic Analysis on Public Bus Transport's Supply Quality by Using AHP, Journal of Transport, 27(3), 268–275.
Gao, Y., Du, X., and Zhang, M. 2011. The Application of Dynamic Priority of AHP on Operation Risk Assessment of Metro. Modeling Risk Management in Sustainable Construction Computational Risk Management. 65-72
Ho, William, 2008. Integrated Analytic Hierarchy Process and Its Applications - A Literature Review, European Journal of Operational Research, 186, 211-228.
Jensen, R.E. 1984. An Alternative Scaling Method for Priorities in Hierarchical Structures, Journal of Mathematical Psychology, 28(3), 317–332.
Jiang J., Chen, Yu-wang, Chen, Ying –wu, and Yang, Ke-wei, 2011. TOPSIS with Fuzzy Belief Structure for Group Belief Multiple Criteria Decision Making International Journal of Expert Systems with Applications, 38(8), 9400-9406.
Lai, Vincent S., Wong Bo K., and Cheung, Waiman, 2002. Group Decision Making in a Multiple Criteria Environment: A Case Using the AHP in Software Selection, European Journal of Operational Research, 137, 134-144.
Li, C.C. and Fun, Y.P., and Hung, J. S. 1997. A New Measure for Supplier Performance Evaluation, Journal of IIE Transactions, 29(9), 753-758.
Lin, C. C., Wang, W, C., Yu, W. D. 2008. Improving AHP for Construction with an Adaptive AHP Approach (A^3), Journal of Automation in Construction, 17(2), 180-187.
Liu, Yun-Ning and Wu, Shiow-yang, Using Normalization and AHP for Improving Supplier Selection Process in Steel Bars Industry, 2013 International Conference on e-Commerce, e-Administration, e-Society, e-Education, and e-Technology, Kitakyushu, Japan, 2013.
Liu, Chih-Ming, Hsu, Hen-Shen, Wang, Shen-Tsu and Lee, Hai-Kun, 2005. A

Performance Evaluation Model Based on AHP and DEA. Journal of the Chinese Institute of Industrial Engineers, 22(3), 243-251.

Lu, Min Hua, Madu, Christian N., Kuei, Chu-hua, and Winokur Dena. 1994. Integrating QFD, AHP and Benchmarking in Strategic Marketing, Journal of Business & Industrial Marketing, 9(1), 41-50.

Rajesh, G. and Malliga, P. 2013. Supplier Selection Based on AHP QFD Methodology, Journal of Procedia Engineering, 64, 1283 -1292.

Rad, B.F., and Abedi, M., An Optimal Load-Shedding Scheme during Contingency Situations Using Meta-Heuristics Algorithms with Application of AHP Method, 2008 11th International Conference on Optimization of Electrical and Electronic Equipment (OPTIM), 167-173, Brasov, 2008.

Searcy, L. D.W., 2004. Aligning the Balanced Scorecard and a Firm's Strategy Using the Analytic Hierarchy Process. Journal of Management Accounting Quarterly. 5(4), 1-10.

Korpela Jukka, Lehmusvaara Antti, and Nisonen Jukka, 2007. Warehouse Operator Selection by Combining AHP and DEA Methodologies, International Journal of Production Economics, 108(1-2), 135-142.

Kurttila Mikko, Pesonen Mauno, Kangas Jyrki, and Kajanus Miika, 2000. Utilizing the Analytic Hierarchy Process (AHP) in SWOT Analysis — a Hybrid Method and Its Application to a Forest-Certification Case, Forest Policy and Economics, 1(1), 41–52.

Kamal M. Al-Subhi and Al-Harbi, 2001. Application of the AHP in Project Management, International Journal of Project Management, 19(1), 19–27.

Ram K Shrestha, Janaki R.R Alavalapati, and Robert S Kalmbacher, 2004. Exploring the Potential for Silvopasture Adoption in South-Central Florida: an Application of SWOT–AHP Method, Agricultural Systems, 81(3), 185–199.

Saaty, T.L. 1980. The Analytic Hierarchy Process, New York: McGraw Hill. International, Translated to Russian, Portuguese, and Chinese, Revised editions, Paperback (1996, 2000), Pittsburgh: RWS Publications.

Saaty, T.L. 1990. How to Make a Decision: The Analytic Hierarchy Process. European Journal of Operational Research, 48, 9-26.

Saaty, T.L. 2003. Decision-making with the AHP: Why is the Principal Eigenvector Necessary. European Journal of Operational Research, 16945, 85-91.

Saaty, T. L. 2008. Decision Making With the Analytic Hierarchy Process. International Journal of Services Sciences, 1(1), 83-98.

Vaidya, O.S. and Kumar, S. 2006. Analytic Hierarchy Process: an Overview of Applications. European Journal of Operational Research, 169, 1-29.

Wu, Hsin-Hung, and Tsai, Ya-Ning, 2012. Using AHP to Evaluate the Criteria of Auto

Spare Parts Industry, International Journal of Quality Quantity, 46, 359-364.

Weiss, E. N. and Rao, V. R. 1987. AHP Design Issues for Large Scale Systems, Journal of Decision Science, 18(1), 43-61, 1987.

Xu, Z.S., Wei, C.P. 2004. A New Method for Priorities in the Analytic Hierarchy Process. Journal of Operational Research Trans. 4(4), 47–54.

Zhang, He, Li Xiu and Liu, Wenhuang, 2006. An AHP/DEA Methodology for 3PL Vendor Selection in 4PL, Computer Supported Cooperative Work in Design II Lecture Notes in Computer Science, 3865, 646-655.